Lesson 01

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Histogram Equalization

Adaptive Histogram Equalization

Contrast Limited Adaptive Histogram Equalization

Histogram Smoothing

Otsu's method

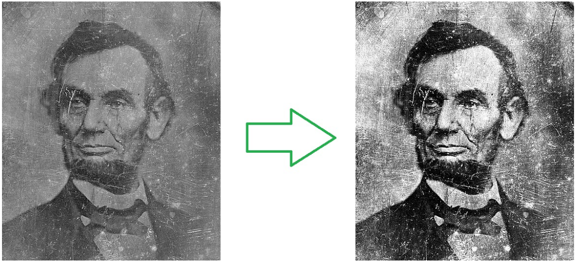
Gaussian Mixture Model

Histogram Equalization

- computer vision method that adjusts the contrast of the image
- criterion is applied on the density of the brightness function
- ordering is maintained

$$T^* = argmin_T(|c_1(T(k)) - c_0(k)|) \quad (1)$$

- where c_0 is the desired cumulative histogram



Obrázek: Histogram equalization

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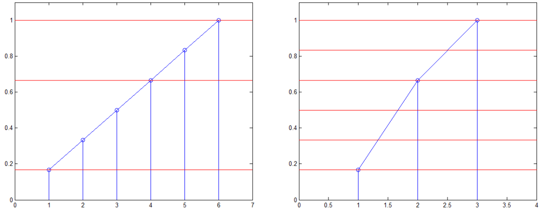
Transformation of random variables

- is used to compute T^*, utilizes cumulative density of the histogram
- example: mapping a dice to $\{1, 2, 2, 2, 3, 3\}$
- $p_{dice} = 1/6 \qquad F_{dice} = \{1/6, 1/3, 1/2, 2/3, 5/6, 1\}$
- $p_{map}(x) = \{1/6, 1/2, 1/3\} \qquad F_{map} = \{1/6, 2/3, 1\}$
- a mapping between F_{dice} and F_{map}

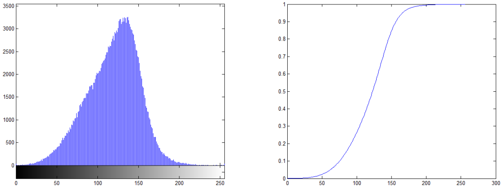


Obrázek: Random Variables transformation

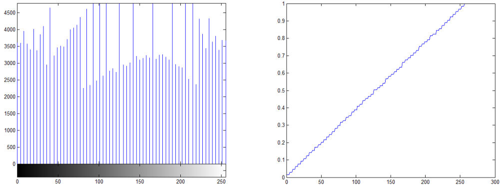
Transformation of random variables

- the transformation has to be monotonic, the ordering has to be maintained
- the mapping $F_{map} \mapsto F_{dice}$ is harder to achieve
- one brightness cannot be divided by the transform
- only translation (mind the ordering!) and merging is possible
- our dice problem results in mapping $\{1 \mapsto 1,\ 2 \mapsto 4,\ 3 \mapsto 6\}$
- we have made use of the whole contrast

Examples



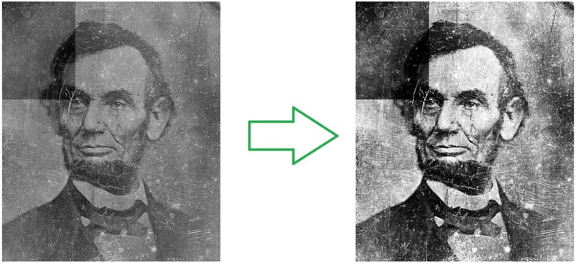
Obrázek: Input histogram and cumulative relative histogram.



Obrázek: Equalized histogram and cumulative relative equalized histogram.

Classic equalization fails

- because the image is handled as a whole, the damage can be seen on the equalized image

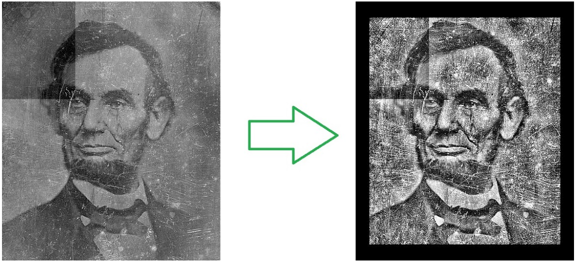


Obrázek: Histogram equalization

- but it also influences the rest of the image

Adaptive Histogram Equalization

- used for images with non-uniform lighting
- the equalization is computed piece-wise



Obrázek: Adaptive Histogram equalization

- problems on the edges of the image and salt & pepper noise
- the size of the window affects the result

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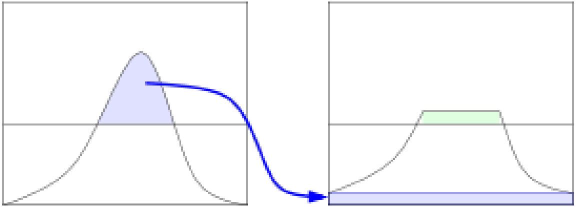
Contrast Limited Adaptive Histogram Equalization

- method that solves the standard equalization problems
- has a parameter of contrast limitation
- it says that no brightness can have a certain count (based on the image size)
- if a brightness exceeds this level, the value is clipped and the remainder is spread across the other brightnesses
- the method does not operate on the pixels directly, but modifies the histogram first and then finds the transform



Obrázek: Contrast Limited Adaptive Histogram equalization

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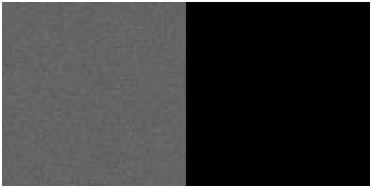
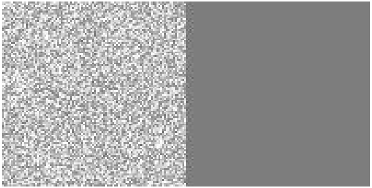
CLAHE examples



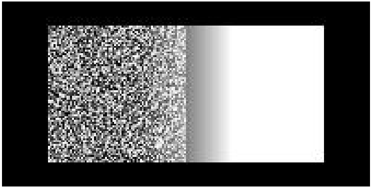
Obrázek: Original image

Obrázek: After Contrast Limited Adaptive Histogram equalization

Classic examples



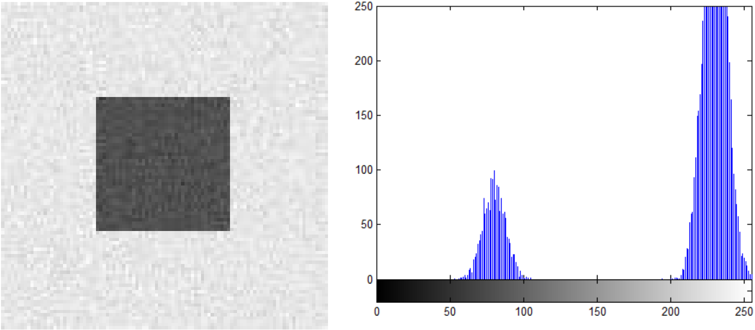
Obrázek: After Histogram equalization



Obrázek: After Adaptive Histogram equalization

Histogram Smoothing

- is used when finding a threshold automatically
- the threshold lies between peaks of a bimodal histogram
- due to the presence of noise we cannot find only "true"peaks
- peak is a local maximum



Obrázek: Input image and its histogram.

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Conventional detection of local maxima

1. If $h'(x) = 0$, then x is an extreme.
2. If $h''(x) < 0$, then x is an local maximum.
3. If $h''(x) > 0$, then x is an local minimum.

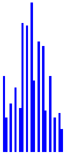
- we usually do not have a function available
- we use approximations

$$h'(x) \approx h(x) - h(x-1) = \triangle h(x), \tag{2}$$

$$h''(x) \approx h(x) - 2h(x-1) + h(x-2), \tag{3}$$

Peaks and noise

- ideal peak - {10, 20, 30, **100**, 30, 20, 10}
- noisy (real) peak - {10, 20, **30**, 20, **50**, 10, **100**, 80, 60}



Obrázek: Noisy peaks.

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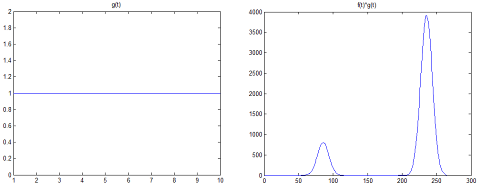
Smoothing as a convolution

- convolution can be used for the purpose of smoothing
- the choice of the type and size of the convolution kernel will affect the result

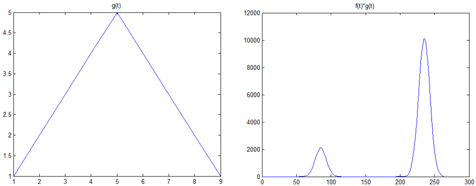
$$(f*g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau = (g*f)(t) \tag{4}$$

$$(f*g)[n] = \sum_{m=-\infty}^{\infty} f[m]g[n-m] \tag{5}$$

Convolution with different kernels



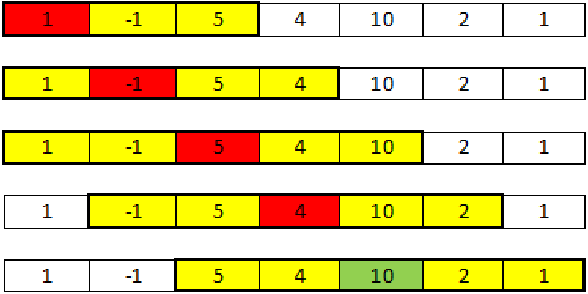
Obrázek: Kernel is a constant



Obrázek: Kernel is a triangle

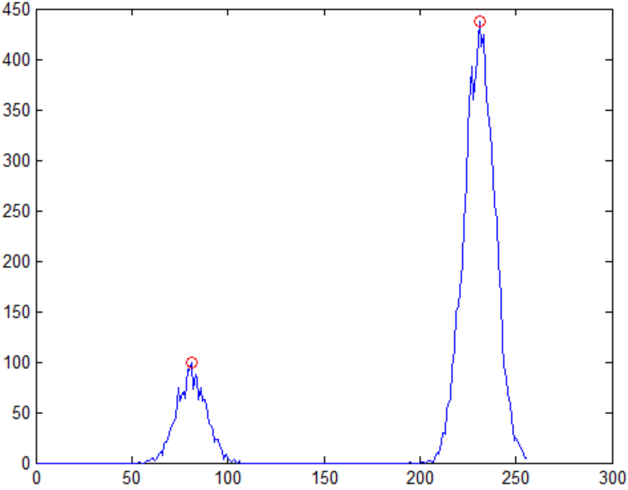
Non-maximum suppression

- easy but powerful tool for the local maxima detection
- uses a local window, the center point is a local maximum if it is the global maximum in the window



Obrázek: Non-maximum suppression

Non-maximum suppression Example



Obrázek: Non-maximum suppression

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Otsu's method

- used for image segmentation
- finds an optimal threshold - a bimodal histogram is desirable
- two classes - $C_0 \in \{1, 2, 3, ..., k\}$, $C_1 \in \{k+1, ..., L\}$

$$p_i = \frac{n_i}{N}, p_i \geq 0, \sum_{i=1}^{L} p_i = 1. \tag{6}$$

$$\omega_0 = Pr(C_0) = \sum_{i=1}^{k} p_i = \omega(k) \tag{7}$$

$$\omega_1 = Pr(C_1) = \sum_{i=k+1}^{L} p_i = 1 - \omega(k) \tag{8}$$

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- definitions of the means of the two classes

$$\mu_0 = \sum_{i=1}^{k} i Pr(i|C_0) = \sum_{i=1}^{k} i \frac{p_i}{\omega_0} = \sum_{i=1}^{k} \frac{ip_i}{\omega_0} = \frac{\mu(k)}{\omega(k)} \tag{9}$$

$$\mu_1 = \sum_{i=k+1}^{L} i Pr(i|C_1) = \sum_{i=k+1}^{L} i \frac{p_i}{\omega_1} = \sum_{i=k+1}^{L} \frac{ip_i}{\omega_1} = \frac{\mu_T - \mu(k)}{1 - \omega(k)} \tag{10}$$

- and the total mean (of the brightness)

$$\mu_T = \mu(L) = \sum_{i=1}^{L} ip_i \tag{11}$$

- definitions of the variances of the two classes

$$\sigma_0^2 = \sum_{i=1}^{k}(i-\mu_0)^2 Pr(i|C_0) = \sum_{i=1}^{k}(i-\mu_0)^2 p_i/\omega_0 \tag{12}$$

$$\sigma_1^2 = \sum_{i=k+1}^{L}(i-\mu_1)^2 Pr(i|C_1) = \sum_{i=k+1}^{L}(i-\mu_1)^2 p_i/\omega_1 \tag{13}$$

- we can proof that the later holds

$$\omega_0\mu_0 + \omega_1\mu_1 = \mu_T, \omega_0 + \omega_1 = 1. \tag{14}$$

- we have to find a criterion to optimize - criteria of discriminative analysis

$$\lambda = \frac{\sigma_B^2}{\sigma_w^2}, \kappa = \frac{\sigma_T^2}{\sigma_w^2}, \eta = \frac{\sigma_B^2}{\sigma_T^2}, \tag{15}$$

$$\sigma_w^2 = \omega_0 \sigma_0^2 + \omega_1 \sigma_1^2 \tag{16}$$

$$\sigma_B^2 = \omega_0(\mu_0 - \mu_T)^2 + \omega_1(\mu_1 - \mu_T)^2 = \omega_0\omega_1(\mu_1 - \mu_0)^2 \tag{17}$$

$$\sigma_T^2 = \sum_{i=1}^{L} (i - \mu_T)^2 p_i \tag{18}$$

- the criteria are dependent (because $\sigma_w^2 + \sigma_B^2 = \sigma_T^2$) , so we can choose only one to optimize

- we choose η because it's the easiest to compute
- the optimal threshold k^* is computed by maximizing η or equally by maximizing σ_B^2

$$\sigma_B^2 = \frac{[\mu_T \omega(k) - \mu(k)]^2}{\omega(k)[1 - \omega(k)]}. \tag{19}$$

$$k^* = \underset{1 \leq k < L}{\operatorname{argmax}} \, \sigma_B^2(k). \tag{20}$$

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Gaussian Mixture Model

- is used to model probability density
- learned via EM

$$gmm = \sum_{i=1}^{N} \alpha_i \mathcal{N}_i(\mu_i; C_i) \tag{21}$$

$$\mathcal{N}_i = \frac{1}{\sqrt{(2\pi)^D |C_i|}} \exp\left(-\frac{1}{2}(x-\mu_i)^T C_i^{-1}(x-\mu_i)\right) \tag{22}$$