Lesson 03

Harris, SIFT, SURF

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Corner detection

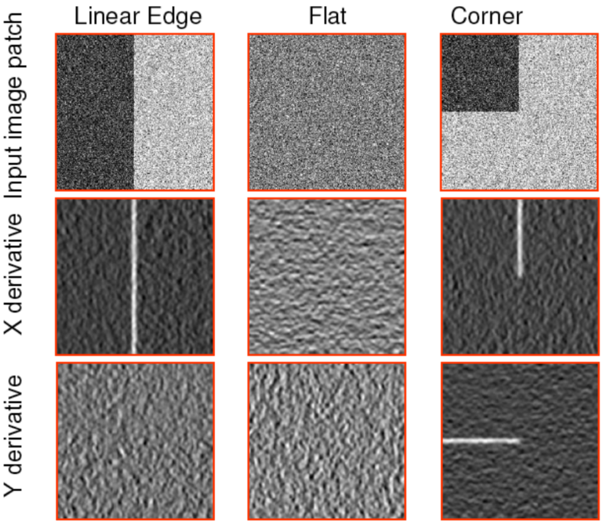
Interest Points

SIFT

SURF

Corner detection

- the corner can be defined as
  1. an intersection of two edges
  2. a (important) point where two dominant directions (gradients) exist
- every corner is an important point, but not the other way around
- a corner detection algorithm needs to be very robust



Obrázek: Different regions and their derivatives.

Moravec corner detection

- one of the first corner detection algorithm
- the alg. tests the similarity of a patch centered on the analyzed pixel with nearby patches
- the similarity is measured as a sum of absolute differences
- the corners are the pixels with a low similarity with its neighborhood - the local maxima of the SoAD

$$E(u,v) = \sum_{x,y} w(x,y)[I(x+u,y+v) - I(x,y)]^2. \quad (1)$$

- $(u,v) = \{(1,0),(1,1),(0,1),(-1,1)\}$

Harris corner detection

- reacts to weak points in Moravec algorithm
- the rectangular window $w(x, y)$ becomes a Gaussian window, which functions also as a filter
- the discretized directions (u, v) disappear and are replaced by Taylor expansion

$$I(x+u, y+v) \approx I(x, y) + I_u(x, y)u + I_v(x, y)v \qquad (2)$$

$$E(u, v) \approx \sum_{x,y} w(x, y)[I_u(x, y)u + I_v(x, y)v]^2. \qquad (3)$$

$$E(u, v) \approx \sum_{x,y} w(x, y)[u^2 I_u^2 + 2uvI_uI_v + v^2 I_v^2], \qquad (4)$$

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- which in matrix form can be written as

$$E(u,v) \approx \sum_{x,y} w(x,y) \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_u^2 & I_u I_v \\ I_u I_v & I_v^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}. \tag{5}$$

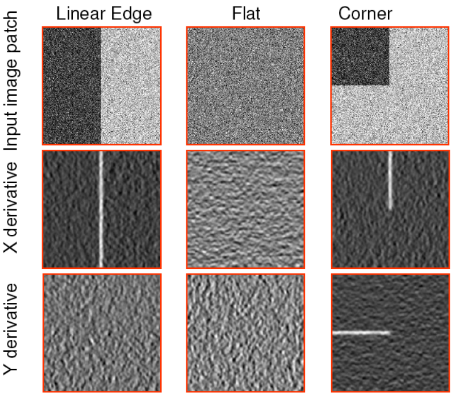
- next we define matrix M

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_u^2(x,y) & I_u I_v(x,y) \\ I_u I_v(x,y) & I_v^2(x,y) \end{bmatrix} \tag{6}$$

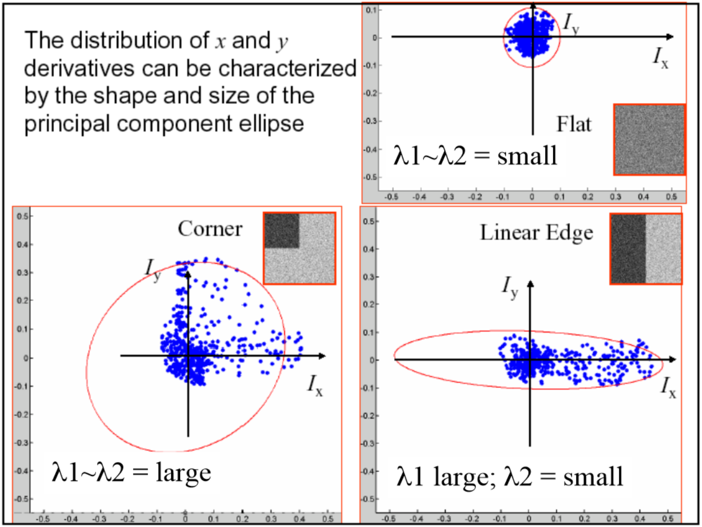
- and then we can write

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}. \tag{7}$$

- the matrix M is called a Harris matrix
- the derivations I_u, I_v can be approximated by gradient operators
- for every pixel we have a matrix M and we analyze their eigenvalues



Obrázek: Different regions and their derivatives.



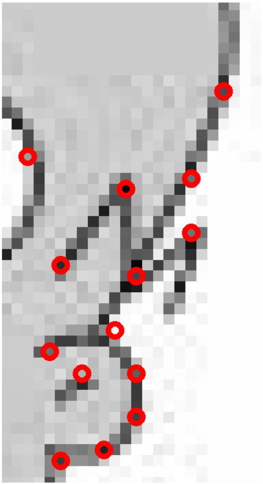
Obrázek: Body proložené elipsami.

- the computational cost of the eigenvalues is very high
- we want the eigenvalues to be relatively the same and also big

$$
\begin{aligned}
\lambda_1 \lambda_2 &= det(M) \\
\lambda_1 + \lambda_2 &= trace(M),
\end{aligned}
\tag{8}
$$

$$
R = det(M) - k(trace(M))^2, \tag{9}
$$

- big $R > 10000$ is a corner
- negative and big $R < -10000$ is an edge
- small $R \in (-10000; 10000)$ is a flat region



Obrázek: Detected corners.

Interest Points

- it has a clear, preferably mathematically well-founded, definition
- it has a well-defined position in image space
- the local image structure around the interest point is rich in terms of local information contents, such that the use of interest points simplify further processing in the vision system
- it is stable under local and global perturbations in the image domain as illumination/brightness variations, such that the interest points can be reliably computed with high degree of reproducibility
- optionally, the notion of interest point should include an attribute of scale, to make it possible to compute interest points from real-life images as well as under scale changes

Scale Invariant Feature Transform

- SIFT is an algorithm that finds interest point
- inspired by Harris corner detection
- the algorithm works the following way:

1. detection of extremes in scale-space representation
2. adjustment of the position of interest points
3. assignment of orientation to the interest points
4. construction of the descriptor of interest point

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Detection of Extremes in Scale-Space

- the scale-space representation is just the image in different resolutions, but with the same width and height
- the different resolution is achieved by convolving the image with a Gaussian kernel

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y),$$
$$\textit{where} \quad G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\left(x^2+y^2\right)}{2\sigma^2}\right). \tag{10}$$

- the Gaussian is self-similar, we can apply it consecutively to obtain more blurred images

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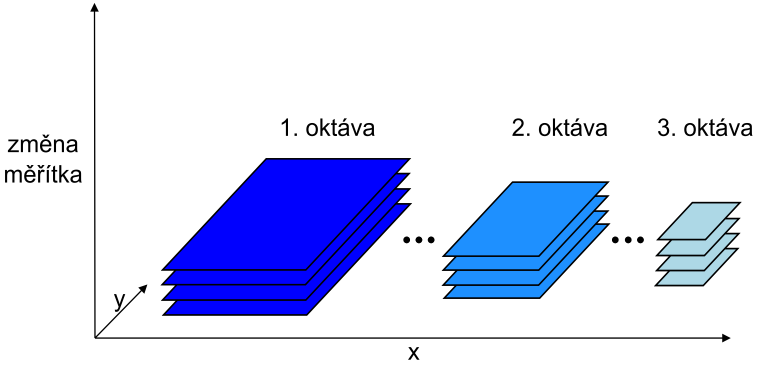
Obrázek: Different scale representations

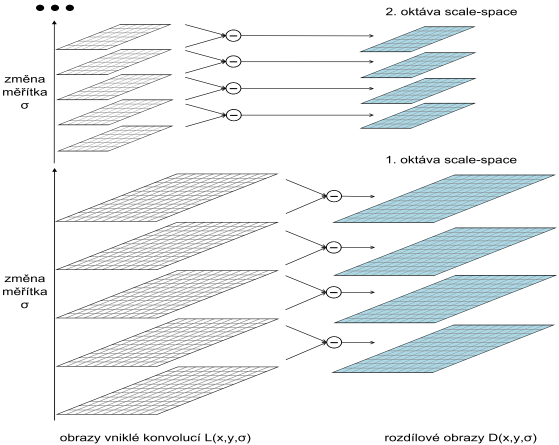
- such images compose an octave
- several octaves are built
- the octave is just the same representation only with smaller width and height

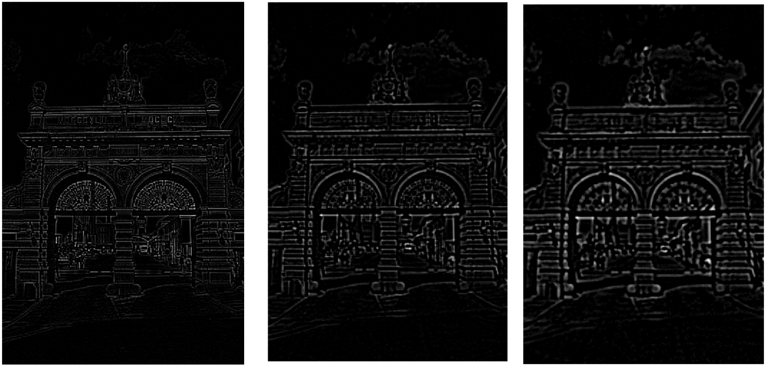


Obrázek: Scale-space representations

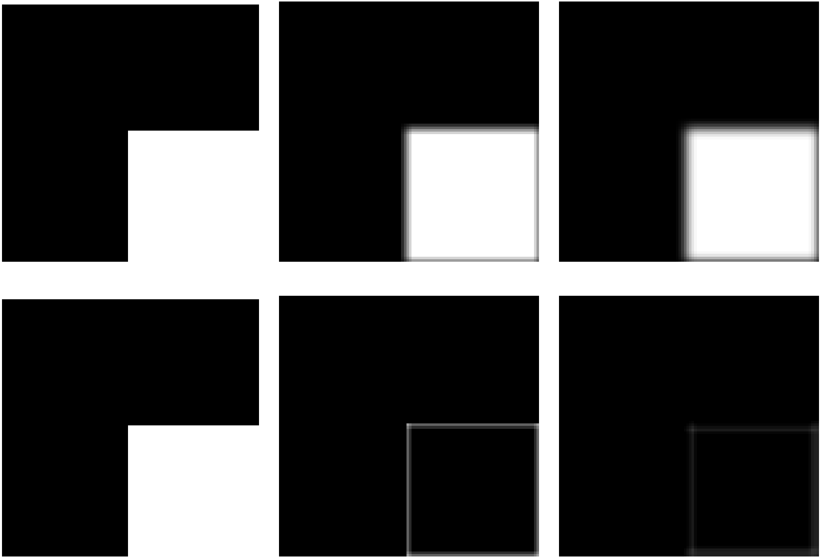
- difference images are constructed by using the octave scale-space representation

$$D(x, y, \sigma) = L(x, y, k\sigma) - L(x, y, \sigma) \tag{11}$$



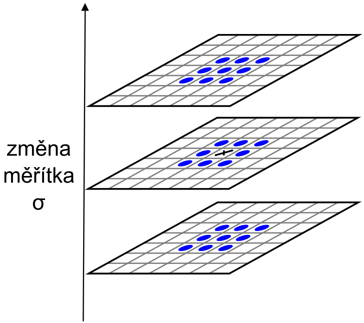


Obrázek: Difference images computed as Difference of Gaussians



Obrázek: Difference images computed as Difference of Gaussians on a corner

- local maxima and minima are detected using non-maxima suppression
- the size of the window is 3x3x3 which means 26 values are compared with the center pixel
- the detected extremes are considered candidates of the interest points



Adjustment of the position of interest points

- the candidate points are fixed on the raster and can be adjusted
- the Taylor expansion is used

$$\tilde{D}(\mathbf{x}) = D + \frac{\partial D^\top}{\partial \mathbf{x}}\mathbf{x} + \frac{1}{2}\mathbf{x}^\top \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x} \tag{12}$$

- the extreme of the expansion is found by derivation and setting the derivative to zero

$$\frac{\partial \tilde{D}}{\partial \mathbf{x}} = \frac{\partial D}{\partial \mathbf{x}} + \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x} \tag{13}$$

$$\hat{\mathbf{x}} = -\left(\frac{\partial^2 D}{\partial \mathbf{x}^2}\right)^{-1} \frac{\partial D}{\partial \mathbf{x}} \tag{14}$$

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Eliminating low-contrast and edge points

- when we use the $\hat{\mathbf{x}}$ to compute the value of $D(\hat{\mathbf{x}})$ we get

$$D\left(\hat{\mathbf{x}}\right) = D + \frac{1}{2}\frac{\partial D^\top}{\partial \mathbf{x}}\hat{\mathbf{x}} \tag{15}$$

- we use the value of $D\left(\hat{\mathbf{x}}\right)$ to eliminate low contrast key-points $(\ < 0.03\ )$
- we also want to eliminate unstable key-points - edge points
- we use similar algorithm as in Harris corner detector - the analysis of eigenvalues of Hess (not Harris) matrix

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix} \tag{16}$$

$$\frac{\mathrm{Tr}\left(\mathbf{H}\right)^2}{\mathrm{Det}\left(\mathbf{H}\right)} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r} \tag{17}$$

$$\frac{\mathrm{Tr}\left(\mathbf{H}\right)^2}{\mathrm{Det}\left(\mathbf{H}\right)} < \frac{(r+1)^2}{r} \tag{18}$$

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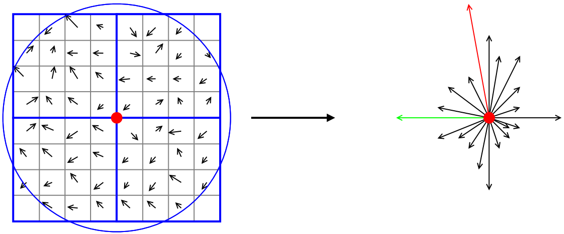
Assigning the orientation to the key-points

- to make the key-points independent on rotation we have to find their "main"orientation
- in the image $L(x, y, \sigma)$ in the key-point we find the magnitudes and directions of the image gradient
- the directions are quantified into bins of 36°



gradienty v okolí

histogram orientací

- if there are more important directions (at least 80% of the biggest) then new key-points are established in the same pixel

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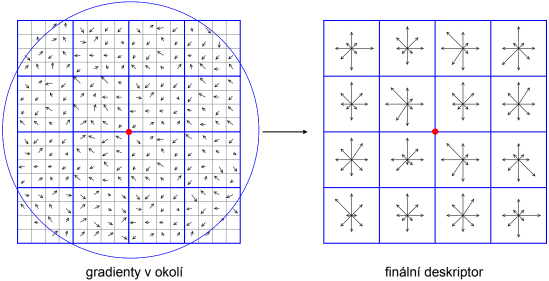
The Key-point descriptor

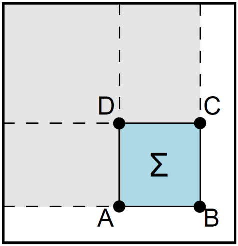
- a description should be independent on geometric and brightness transformations
- the neighborhood of the key-point is divided into 4x4 regions
- in each region the gradients are computed
- the orientations of the gradients are then rotated to align with the dominant direction
- they are concatenated into a 128-dimensional feature vector



SURF - Speeded Up Robust Features

- inspired by SIFT with real-time capabilities
- the DoG images and computing of Hess matrix is integrated into computing the determinant of Hess matrix
- this approach is using the integral image

$$I_{\Sigma}(x,y) = \sum_{i=0}^{i \leq x} \sum_{j=0}^{j \leq y} I(i,j) \tag{19}$$



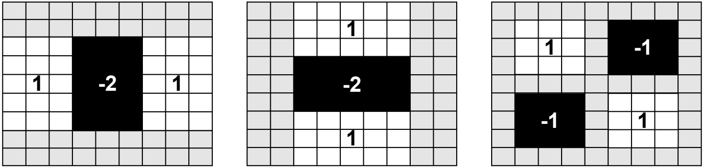
Σ = B - A - C + D

Hess matrix approximation

- the Hess matrix can be written as

$$H(x,y,\sigma) = \begin{bmatrix} L_{xx}(x,y,\sigma) & L_{xy}(x,y,\sigma) \\ L_{yx}(x,y,\sigma) & L_{yy}(x,y,\sigma) \end{bmatrix} \tag{20}$$

- the approximation uses discrete convolution with kernels



- the determinant of Hess matrix is then computed as

$$\mathrm{Det}(\mathbf{H}_{aprox}) = D_{xx}D_{yy} - (wD_{xy})^2 \tag{21}$$

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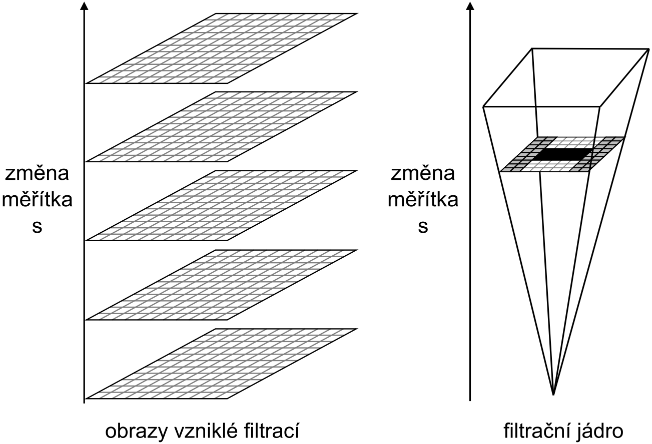
Scale-space approximation

- the scale-space does not need to be constructed explicitly
- different sizes of the kernels fulfill this operation



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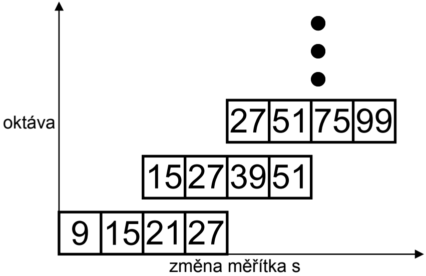
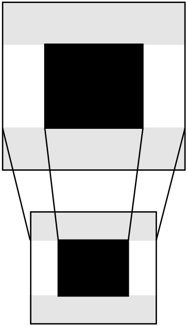
- the different octaves are constructed by using different combinations of sizes of the kernels



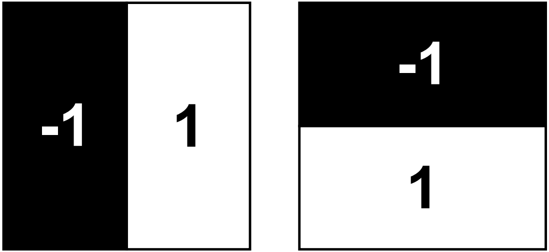


Obrázek: Změny rozměrů filtračních jader pro jednotlivé oktávy scale-space (vlevo) a názorná ukázka změny rozměru jádra (vpravo). Poznamenejme, že krok l_0 je vždy sudý (6, 12, 24) tak, aby při zvyšování měřítka nedocházelo ke změně struktury filtračních jader.

- again, the key-points are local extremes of the determinants of Hess matrix

Orientation of the Key-points

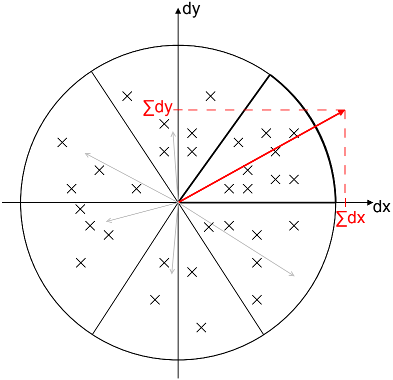
- the Haar filters are used to approximate the orientation of the gradients
- the size of the filters is relative to the scale (4σ) at which the key-point is detected



Obrázek: Haarova vlnka aproximovaná obdélníkovými filtry ve směru osy x a y.

- the responses are filtered with a Gaussian

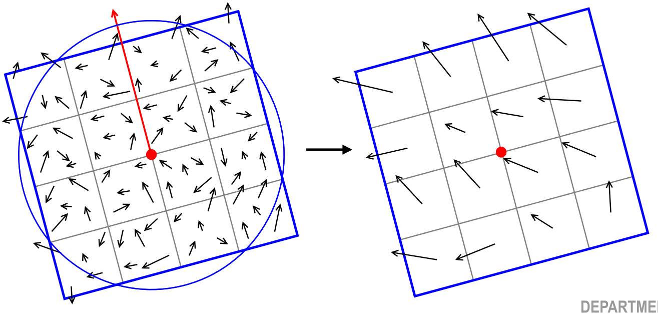
- the space of the responses (d_x, d_y) is divided into several segments



- the dominant direction is the one with the biggest sum of vectors inside it

The SURF descriptor

- a neighborhood around the key-point is constructed and rotated by the angle of the dominant direction
- the neighborhood is of size 20σ
- this patch is divided into 4×4 segments
- for each segment the responses of the Haar filter is computed - (d_x, d_y)
- the descriptor is then a vector $(\sum d_x, \sum d_y, \sum |d_x|, \sum |d_y|)$



Application of key-points

- https://www.youtube.com/watch?v=-r9J1eO4qg4